SHRI JAIN PUBLIC SCHOOL, BIKANER
HOLIDAY HOMEWORK
(2017–18)

CLASS – XII [SCI.]

FOLLOW YOUR DREAM

Follow your dream.
Take one step at a time and don't
settle for less,
Just continue to climb.
Follow your dream.
If you stumble, don't stop and lose
sight of your goal
Press to the top.
For only on top can we see the whole view
Can we see what we've done and
what we can do;
Can we then have the vision to seek
something new,
Press on.
Follow your dream

Summer Vacation is synonymous with fun, frolic, getting up late in the morning, playing for longer hours with friends, going for picnics, exploring new places and watching fun filled shows on television. But dear children, there is a lot more you can do to make your vacations more interesting, meaningful and full of fun. It will surely prepare you for a better and more fruitful year ahead.

We have planned some interesting activities for you. So get ready to enjoy your summer vacation!

Note:–

1. Read newspaper and listen to news-headlines to keep yourself updated.
2. Stay indoors in the afternoon.
3. Help your parents in household chores.
4. Go for morning and evening walk.
5. Be motivated as you go through the poem "***FOLLOW YOUR DREAM***..

- All the Assignments for Summer Holidays should be done in Class work copy of the respective subject.
- Written work should be done with neat and clean handwriting.

MATHS

- Do the **WORKSHEETS** given at the end.

PHYSICS

- Do the **NCERT** examplers of Chapter 1 and 2.
- Read the Chapter 1 and 2.
- Practices all the diagrams and numerals from these chapters.

CHEMISTRY

- Do the **WORKSHEETS** given at the end.

BIOLOGY

- Do the **WORKSHEETS** given at the end.

LEARN TO BE RESPONSIBLE AND DEVELOP PERSONAL SKILLS:

- Take up one task everyday and figure out how you will complete this task. For example
- Keep things in their proper places so as to keep your bedroom clean
- Help your mother in laundary
- Water the plants.
- Dust and clean your room.
- Look after your younger brother or sister
- Set the dining table for your family
- Feed your pet if you have one

DEVELOP SOME SOCIAL SKILL:

- Give respect to the elders and love to the young ones.
- Four magic words that are basics of good manners– Please, Thank you, Excuse me and Sorry. Make these four words a habit and see the difference.
- Wish and welcome the guests
- Converse with your grandparents
- Go for a heritage walk

DEVELOP PERSONAL HYGIENE:

- Keep your surroundings clean
- Trim your nails once a week
- Early to bed, early to rise, Plenty of sleep helps you concentrate.
- Wash your hands before and after meals.
- Wash your hair and keep it neat by combing
- Bath or shower daily. Wash hands after using the toilet.
- Eat a healthy and balanced diet.

Biology assignment
Class 12 (holidays home work) 2017-2018

Chapter 1

VSA (1 Mark)

1. Offsprings produced by asexual reproduction are referred to as clones. Why?
2. Name the most invasive aquatic plant weed which is called as 'Terror of Bengal'.
3. How does Zygote usually differ from Zoospore in terms of ploidy?
4. Mention the main difference between the offspring produced by asexual reproduction and progeny produced by sexual reproduction.
5. There are 380 chromosomes in meiocytes of a butterfly. How many chromosomes do male gamete of butterfly have?
6. Which characteristic property of Bryophyllum is exploited by gardeners and farmers?
7. Mention the unique flowering phenomenon exhibited by *strobilanthes kunthiana* (Neelakuranji).
8. Mention the unique feature with respect to flowering and fruiting in bamboo species.

SA I (2 Marks)

9. Higher organisms have resorted to sexual reproduction inspite of its complexity. Why?
10. Tapeworms posses both male and female reproductive organs. What is the name given to such organism? Give two more examples of such organisms.
11. Study the relationship between first two words and suggest a suitable word for fourth place.
    (a) Male flower : Stamens :: Female Flower : .....
    (b) Birds : oviparous :: Primates : .....
    (c) Chlamydomonas : Zoospores :: Penicilium : .....
    (d) Ginger : Rhizome :: Agave : .....
12. Bryophytes and Pteridophytes produce a large number of male gametes but relatively very few female gametes. Why?

Chapter 2- sexual reproduction in plants

VSA (1 MARK)

1. In a young anther, a group of compactly arranged homogenous cells were observed in the centre of each microsporangium. What is the name given to these cells?
2. Give the scientific name of a plant which came to India as a contaminant with imported wheat and causes pollen allergy.
3. Pollen grains of water pollinated species have a special characteristic for protection from water. What is that?
4. Why are pollen grains produced in enormous quantity in Maize?
5. In same species of Asteraceae and grasses, seed are formed without fusion of gametes. Mention the scientific term for such form of reproduction.

6. Arrange the following in correct developmental sequence: Male gamete, Potential pollen mother cell, sporogenous tissue, Pollen grains, Microspore tetrad.

7. If the diploid number of chromosomes in an angiospermic plant is 16. Mention number of chromosomes in the endosperm and antipodal cell.

SA-I (2 MARKS)

8. In angiospermic plant before formation of microspore sporogenous tissue undergo cell division
   (a) Name the type of cell division.
   (b) What would be the ploidy of the cells of tetrad?
9. Outer envelop of pollen grain made of a highly resistant substance. What is that substance? At which particular point the substance is not present?
10. Fruits generally develops from ovary, but in few species thalamus contributes to fruit formation.
    (a) Name the two categories of fruits.
    (b) Give one example of each.
11. Among the animal, insects particularly bees are the dominant pollinating agents. List any four characteristic features of the insect pollinated flower.
12. Differentiate between geitonogamy and xenogamy.
13. Even though each pollen grain has two male gametes. Why are at least 10 pollen grains and not 5 pollen grains required to fertilise 10 ovules present in a particular carpel?

VSA [1 MARKS]

1. What do you mean by variations and give its types.
2. To identify either F_1 generation is homozygous or heterozygous which cross would you suggest & make such cross
3. Define the following terms:
   (a) Allele (b) Genotype
4. What do you mean by Co-dominance give its examples.
5. What technique was used by Mendel to perform his experiments.

SA [2 MARKS]

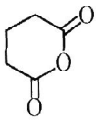
1. What do you mean by inheritance of two genes and make such a cross.
2. What do you mean by multiple allelism and give its examples.
3. If red flowers are crossed with white flowers than in F1 generation pink flowers were obtained. Which type of interaction it was? Explain it by making such cross.
4. What do you mean by chromosomal theory of sex determination and explain its xx-xy type method.
5. Explain following interactions:
   (a) Complementary gene (b) collaboratory gene

SHRI JAIN PUBLIC SCHOOL

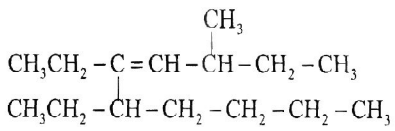
Bhagwan Mahaveer Marg, Gangashahar Road, Bikaner (Raj.)
Ph. 0151-2202906, 9214204576

Chemistry (Class-XII)
(Holiday Homework)

1. The IUPAC name of the compound
   (1) 3-thyl-3-methylhex-1-yne (2) 4-methyl-4-ethynlhex-4-yne
   (3) 3-mthyl-3-propyl pent-1,4-diyne (4) 3-etthnyl-3-methylhx-1-yne
2. The IUPAC name of $C_6H_5COCHOHC_6H_5$ is :
   (1) Benzoylbenzyl alcohol (2) 1-Hydroxy-1,2-diphenylethane-2-one
   (3) 2-hydroxy-1, 2-diphenylethane-1-one (4) Benzoyl phenyl carbinol
3. The correct name for sym-trichloroisobutane is :
   (1) tri(chloromethy;) methane (2) tris(chloromethyl) methane
   (3) 1,3-dichloro-2-(chloromethyl)ptopane (4) 1,1-Do(chloromethyl)-2-chloroethane
4. The IUPAC name for the compound $(CH_3CH_2CH_2CH_2)_2C = C = O$ is :
   (1) dibutylketene (2) butyldene methanone
   (3) 2-butylhex-1-en-1-one (4) 2,2-dibutyleth-1-en-1-one
5. Number of heteroatoms (other than C) present in the following heterocyclic compound



   is :
   (1) 3 (2) 2 (3) 1 (4) 0
6. The IUPAC name of $\overset{|}{CH} = \overset{|}{CH}$ (CHO, NH_2) is :
   (1) 1-amino prop-2-enal (2) 3-amino prop-2-enal
   (3) 1-amino-2-formulethene (4) 3-amino-1-oxoprop-2-ene
7. IUPAC name of C_6H_5CN is :
   (1) Phenyl nitrile (2) Benzenecarbnitrile
   (3) Benzyl nitrile (4) Phenyl cyanide
8. The IUPAC name of the compound $CH_3COCH(Cl) - CH(I) - COOH$ is :
   (1) 2-iodo-3-chloro-4-pentanoic aicd (2) 4-oxo-3-chloro-2-iodo pentanoic acid
   (3) 4-carboxy-4,3-chloro-2-butanonc (4) 3-chloro-2-iodo-4-oxo-pentanoic acid
9. IUPAC name of propargyl alcohol is :
   (1) prop-2-yn-1-ol (2) prop-2-en-lol (3) prop-1-en-2-ol (4) prop-1-yn-2-ol
10. The correct IUPAC name of the compound is :



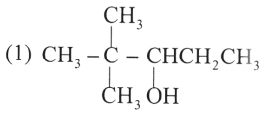
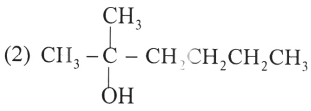
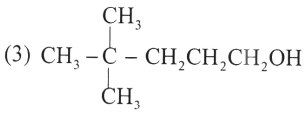
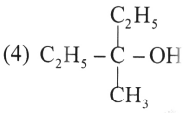
    (1) 5,6-diethyl-8-methyldec-6-ene (2) 5,6-diethyl-3-methyldec-4-ene
    (3) 6-butyl-5-ethyl-3-methyloct-4-ene (4) 2,4,5-triethyl-3-ene
11. The IUPAC name of isoprene is :
    (1) 1,3-butadiene (2) 2-methyl-1,3-buadiene
    (3) 2-methyl-1-butene (4) 3-methyl-1-butene

12. IUPAC name of mesitylene is :
(1) 1,2,3-trimethyl benzene (2) 1,3,5-trimethyl benzene
(3) 1,2,4-trimethyl benzene (4) 1,4-dimethylbenzene

13. Which of the following statement is incorrect for the homologous series of alkanes :
(1) All are straight chain compounds (2) Have the general formula C_nH_{2n+2}
(3) Have similar chemical properties (4) Show a regular gradation of physical properties

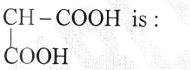
14. Which of the following forms a homologous series :
(1) Ethene, ethylene, acetylene (2) Ethane, propane, butanone
(3) Methanal, ethanol, propanoic acid (4) butane, 2-methylbuane, 2,3-dimethylbutane

15. Neoheptyl alcohol is correctly represented as :

(1) 
(2) 
(3) 
(4) 

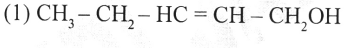
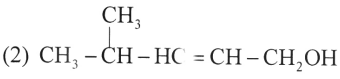
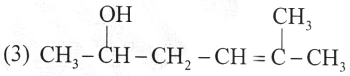
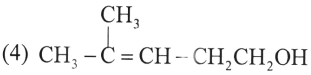
16. The common name of $(CH_3)_3CC_2H_5$ is :
(1) isohexane (2) neohexane (3) trimethylpropane (4) none is correct

17. The family to which methoxyethene belongs :
(1) Hydrocarbon (2) Ketone (3) Unsaturated ether (4) Ester

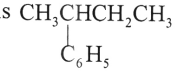
18. The correct IUPAC name of  is :
(1) tricarboxymethane (2) propanetrioic acid
(3) tributanoic acid (4) methanetricarboxylic acid

19. The lowest lakane which has ethyl group as substituent has IUPAC name :
(1) 2-ethylpropane (2) 2-ethylbutane (3) 3-ethylpentane (4) None of these

20. The stucture of 4-methylpent-2-en-1-ol is :

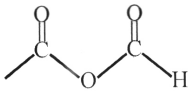
(1) 
(2) 
(3) 
(4) 

21. The IUPAC name of the hydrocarbon $CH \equiv CCH = CH - CH = CH_2$ is :
(1) hex-3,5-dien-5-yne (2) hex-1,2-dien-1-yne
(3) hexa-1,3-dien-5-yne (4) hexa-3,5-dien-1-yne

22. The IUPAC name of the following compound is 
(1) 2-cyclohexylbutane (2) sec-butylbenzene
(3) 3-cyclohexylbutane (4) None of these

23. The systematic name of $C_{17}H_{35}COOH$ is :
(1) heptadecanoic acid (2) octadecanoic acid
(3) steric acid (4) palmitic acid

24. The correct IUPAC name of the following is :



(1) butane-2,4-dione
(2) formyl ethanoate
(3) acetic anhydride
(4) ethanoicmethanoic anhydride

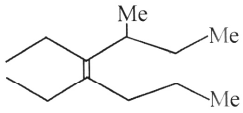
25. IUPAC name of citic acid is :
(1) 1,2,3-Tricarboxy-2-propanal
(2) 3-hydroxy-1,2,3-ticarboxy propanal
(3) 2-hydroxypropane-1,2,3-tricarboxylic acid
(4) 1,2,3-tricarboxylic acid

26. IUPAC name of  :

(1) 4-methyl cyclohexene
(2) 1-methyl cyclohex-3-ene
(3) 1-methyl cydohex-4-ene
(4) Methyl cyclohexene

27. The structure of methyl acrylate is :
(1) CH_3–CH=CNCO_2CH_3
(2) CH_3COOCH_3
(3) CH_2=CH–$CO_2C_2H_5$
(4) CH_2=CHCO_2CH_3

28. CH_3–COOOCCH=CH_2 its name is :
(1) Pentenoic anhydride
(2) Acetic acrylic anhydride
(3) Acetyl acryloside
(4) All of these

29.  its IUPAC name is :

(1) 5-methyl-4, 5-diethyl-4-octene
(2) 4, 5-diethyl-3-methyl-4-octene
(3) 4, 5-diethyl-5-methyl-3-octene
(4) 4, 5-diethyl-5-methyl-5-octene

30. Consider the following :

I. $CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-\underset{\displaystyle CH_3}{\underset{|}{CH}}-CH_3$

(a) 4-chloro-3-hydroxybutanoic acid
II. $CH_2=CH-CH_2-CH_2-CH_2OH$
(b) ethanedial
III. CHO–CHO
(c) 4-pentene-1-ol
IV. CH_2Cl–CHOH–CH_2COOH
(d) 2, 3-dimethylbutane
Correct matching is :
(1) I-d, II-c, III-b, IV-a
(2) I-c, II-b, III-a, IV-d
(3) I-b, II-a, III-c, IV-d
(4) I-a, II-b, III-c, IV-d

31. Which of the following class of compounds can ? Show metamerism :
(1) Ethers
(2) Ketones
(3) 2° amines
(4) Nitriles

32. Amines exhibit :
(1) Chain and position isomerism
(2) Functional isomerism
(3) Metamrism
(4) All of the above

33. o-chloro toluene and benzyl chloride are :
(1) Geometical isomerism
(2) Functional
(3) Chain
(4) Metamerism

34. This structure is similar to ☐ :

(1) $CH_2 = CH - CH_2CH_3$ (2) $CH_3 - CH = CH - CH_3$

(3) $CH_3 - CH_2 - C \equiv CH$ (4) C_4H_{10}

35. $CH_3CH_2 - \underset{\underset{OH}{|}}{CH} - CHO$ and $CH_2CH_2CH_2COOH$ are :

(1) Position isomers (2) Functional isomers

(3) Metamerism (4) Tautomerism

36. $CH_3OCH_2CH_2CH_3$ and $CH_3-O-CH\begin{matrix}CH_3\\CH_3\end{matrix}$ are :

(1) Chain isomers (2) Tautomers (3) Position isomers (4) Functional isomers

37. Metamerism is shown by :

(1) Diethyl ether and n-propyl methyl ether (2) Ethyl alcohol and diethyl ether

(3) Acetone and propionaldehyde (4) Propionic acid and acetic acid

38. Which of the following pair is rightly matched :

(A) Functional isomer of 2-butyne 1, 3-butadiene

(B) Position isomer of butanal 2-methyl propanal

(C) Metamer of methyl acetate Propionic acid

(D) Chain isomer of isobutylene 1-butene

(1) A and B (2) B and C (3) C and D (4) A and D

39. Of the four compounds :

(a) $CH_3 - \underset{\underset{OH}{|}}{CH} - CH_2 - CHO$ (b) $CH_3 - CH_2 - CH_2 - \underset{\underset{O}{\|}}{C} - OH$

(c) $\underset{\underset{OH}{|}}{CH_2} - CH_2 - CH_2 - CHO$ (d) $CH_3 - CH_2 - \underset{\underset{O}{\|}}{C} - OCH_3$

[a and c] show.....isomerism, [a and b] show.....isomerism and [b and d] show.....

(1) Position, Functional, Functional (2) Metamerism, Chain, chain

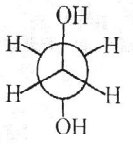
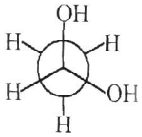
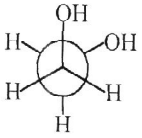
(3) Chain, Functional, Functional (4) Position, Position, Functional

40. Molecular formula $C_6H_{14}O$ will show :

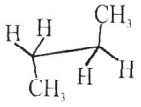
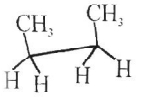
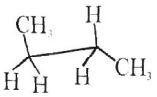
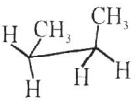
(1) Functional isomerism (2) Chain isomerism

(3) Metamerism (4) All of these

41. Which of the following conformers for ethylene glycol is most stable :

(1)  (2)  (3)  (4) None

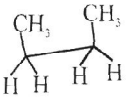
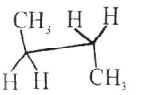
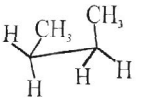
42. Which of the following sawhorse projection formulae represents the gauche conformation of butane :

(1)  (2)  (3)  (4) 

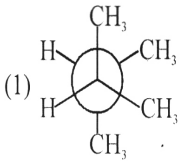
43. How many gauche conformations are possible for n-butane :

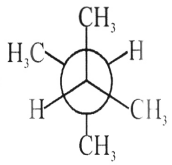
(1) 2 (2) 3 (3) 4 (4) 1

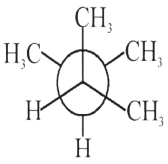
44. Which of the following conformers of n-butane represents anti staggered conformation :

(1)  (2)  (3)  (4) None of these

45. The most stable conformation of 2, 3-dimethyl butane is :

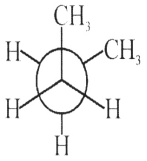
(1) 

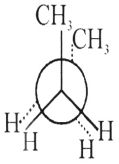
(2) 

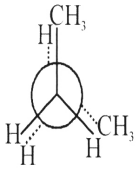
(3) 

(4) None

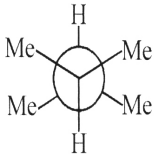
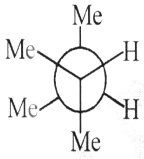
46. Which of the following conformers of n-butane has torsional strain :

(1) 

(2) 

(3) 

(4) Both (2) and (3)

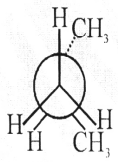
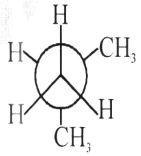
47.  and  are :

(1) Conformational isomers
(2) Configurational isomers
(3) Constitutional isomers
(4) Identical

48. In n-butane anti form is at :

(1) 160° (2) 180° (3) 120° (4) 0°

49. What are the type of isomers in following pairs :

 and 

(1) Conformers (2) Position (3) Optical (4) Geometrical

50. Which of the following is the correct order of stability of the following four distinct conformations of n-butane :

(1) staggered > gauche > partialy eclipsed > fully eclipsed
(2) gauche > staggered > partialy eclipsed > fully eclipsed
(3) staggered > partialy eclipsed > gauche > fully eclipsed
(4) fully eclipsed > staggered > partialy eclipsed > gauche

CHAPTER 5 : CONTINUITY

ASSIGNMENT-1

1. The function f is defined as

$$f(x) = \begin{cases} x^2 + ax + b & 0 \le x < 2 \\ 3x + 2 & 2 \le x \le 4 \\ 2ax + 5b & 4 < x \le 8 \end{cases}$$

If function is continuous on [0, 8], find the values of a and b.

2. Discuss the continuity of the function f given by
$f(x) = |x - 1| + |x - 2|$ at $x = 2$

3. Examine the continuity of the function f given by

$$f(t) = \begin{cases} \dfrac{\cos t}{\frac{\pi}{2} - t}; & t \ne \dfrac{\pi}{2} \\ 1; & t = \dfrac{\pi}{2} \end{cases} \text{ at } t = \frac{\pi}{2}$$

4. Let $f(x) = \begin{cases} \left\{\dfrac{1 - \cos 4x}{x^2}\right. & x < 0 \\ a & x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4} & x > 0 \end{cases}$ Determine the value of a so that f is continuous at $x = 0$.

(NCERT Exemplar CBSE 2010, 2012)

5. Determine the values of a, b, c for which the function given by

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & x < 0 \\ c & x = 0 \\ \dfrac{\sqrt{x + bx^2} - \sqrt{x}}{bx^{3/2}} & x > 0 \end{cases}$$

is continuous at $x = 0$.

6. (i) If $f(x) = \begin{cases} \dfrac{x-4}{|x-4|} + a & x < 4 \\ a + b & x = 4 \\ \dfrac{x-4}{|x-4|} + b & x > 4 \end{cases}$ is continuous at $x = 4$, find a and b.

(ii) Determine if f defined by $f(x) = \begin{cases} x^2 \sin\frac{1}{x}; & x \neq 0 \\ 0; & x = 0 \end{cases}$ is a continuous function?

7. If $f(x) = \begin{cases} \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1}-1}; & x \neq 0 \\ K; & x = 0 \end{cases}$ is continuous at x = 0, find K.

8. Find the value of 'a' for which the function f defined by

$f(x) = \begin{cases} a\sin\frac{\pi}{2}(x+1); & x \leq 0 \\ \frac{\tan x - \sin x}{x^3}; & x > 0 \end{cases}$ should be continuous at x = 0. (CBSE 2011)

9. For what value of K is the following function continuous at x = 2

$$f(x) = \begin{cases} 2x+1; & x < 2 \\ K; & x = 2 \\ 3x-1; & x > 2 \end{cases}$$ (CBSE 2008)

10. $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x}; & x < \frac{\pi}{2} \\ a; & x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2}; & x < \frac{\pi}{2} \end{cases}$

If f is continuous at $x = \frac{\pi}{2}$, find a and b.

11. If the function f defined below is continuous at x = 0, find the value of K:

$$f(x) = \begin{cases} \frac{1-\cos 2x}{2x^2} & x < 0 \\ K & x = 0 \\ \frac{x}{|x|} & x > 0 \end{cases}$$ (CBSE 2010)

12. Show that the function defined by g(x) = x – [x] is discontinuous at all integral points. Here [x] denotes the greatest integer less than or equal to x.

13. In the following, determine the value of constant involved in the definition so that the given function is cotinuous:

(i) $f(x)=\dfrac{\sqrt{1+px}-\sqrt{1-px}}{x}$ $-1\le x<0$

$\dfrac{2x+1}{x-2}$ $0\le x\le 1$

(ii) $f(x)=\begin{cases}\dfrac{K\cos x}{\pi-2x} & x<\dfrac{\pi}{2}\\ 3 & x=\dfrac{\pi}{2}\\ \dfrac{3\tan 2x}{2x-\pi} & x>\dfrac{\pi}{2}\end{cases}$ (CBSE 2010)

14. $f(x)=\dfrac{\sqrt{2}\cos x-1}{\cot x-1}$ $x\neq\dfrac{\pi}{4}$

Find the value of $f\left(\dfrac{\pi}{4}\right)$ so that f becomes continuous at $x=\dfrac{\pi}{4}$.

15. Show that the function f given by

$$f(x)=\begin{cases}\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} & x\neq 0\\ 0 & x=0\end{cases}$$

is discontinuous at x = 0.

16. Let $f(x)=\begin{cases}\dfrac{2^{x+2}-16}{4^x-16} & \text{if } x\neq 2.\end{cases}$

For what value of k, f is continuous at x = 2.

17. Show that the function f(x) = |sin x + cos x| is continuous at $x=\pi$.

18. Check the differentiability of cos |x| on R.

Assignment 1(Answers)

1. a = 3, b = – 2

4. a = 8

5. $a=\dfrac{-3}{2}, b\in R-\{0\}, c=\dfrac{1}{2}$

6. (i) a = 1, b = – 1

7. $k = -4$

8. $a = \frac{1}{2}$

9. $K = 5$

10. $a = \frac{1}{2}$ $b = 4$

11. $K = 1$

13. (i) $p = -\frac{1}{2}$ (ii) $K = 6$

14. $\frac{1}{2}$

16. $K = \frac{1}{2}$

18. Continuous every where

CHAPTER 1 : RELATIONS AND FUNCTIONS

ASSIGNMENT
VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Let the sets A and B consists of 3 and 5 elements respectively. Find the total number of functions from A to B. How many of them are one-one?
2. If R, be a relation in set N given by
   $R = \{a, b) : a = b - 3, b > 5\}$
   Does elements $(5, 7) \in R$?
3. If $f : \{1, 3) \to \{1, 2, 5\}$ and $g : \{1, 2, 5\} \to \{1, 2, 3, 4\}$ be given by
   $f = \{(1, 2), (3, 5)\}$, $g = \{(1, 3), (2, 3), (5, 1)\}$
   Write down gof.
4. Let $g, f : R \to R$ be defined by
   $g(x) = \frac{x+2}{3}$, $f(x) = 3x - 2$. Write fog.
5. If $f : R \to R$ defined by
   $f(x) = \frac{2x-1}{5}$
   be an invertible function, write $f^{-1}(x)$.
6. If $f(x) = \frac{x}{x+1}$ $\forall$ $x \neq -1$, Write f of (x).
7. Let * be a Binary operation defined on R, then if
   (i) $a * b = a + b + ab$, write $3 * 2$
   (ii) $a * b = \frac{(a+b)^2}{3}$, Write $(2 * 3) * 4$.
   (iii) $a * b = 4a - 9b^2$, Write $(1 * 2) * 3$.
8. What is the number of bijective function from a set A to B, when A and B have same number of elements?
9. If $f, g : R \to R$ be defined by
   $f(x) = \frac{3x-7}{8}$, $g(x) = \frac{8x+7}{3}$, then
   What is fog (7).
10. What is the smallest equivalence relation defined on the set {a, b, c}?
11. If A = {1, 2, 3, 4} and B = {5, 6, 7}, find the number of functions which can be defined from A to B.
12. If f be the greatest integer function and g be the modulus function, then find the value of

$$\text{gof}\left(\frac{-15}{4}\right) - \text{fog}\left(\frac{-15}{4}\right).$$

13. Suppose '*' is the binary operation defined on Z, the set of all integers, defined as $a * b = a + b + 1$, $\forall\ a, b \in Z$, find the identity element w.r.t. this operation.

14. Let A = {a, b}, find the number of binary operations that can be defined on A.

15. For a and $b \in R$, defined $a * b = \frac{a}{a+b}$, where $a + b \neq 0$. If $a * b = 5$, find the value of $b * a$.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

16. Determine whether each of the following relations are:
    (i) Symmetric (ii) Transitive (iii) Equivalence relation.
    (a) $R_4 = \{(a, b) : 3a - b = 0, a, b \in R\}$
    (b) $R_5 = \{(a, b) : a \leq b^3, a, b \in R\}$
    (c) $R_6 = \{(a, b) : a = b + 2, a, b \in R\}$

17. Check the following functions for one-one and onto.
    (a) $f : R \to R$, $f(x) = |x - 3|$
    (b) $f : R \to R$, defined by $f(x) = \sin x$

18. Let $f : R \to R$ be a function defined by $f(x) = x^3 + x$. Show that f is invertible.

19. Let $f : R - \left\{\frac{-4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ be a function given by $f(x) = \frac{4x}{3x+4}$. Show that f is invertible with $f^{-1}(x) = \frac{4x}{4-3x}$.

20. If $A = N \times N$ and binary operation * is defined on A as $(a, b) * (c, d) = (ac, bd)$.
    (i) Check * for commutativity and associativity.
    (ii) Find the identity element for * in A (If exists).

21. Show that the relation R defined by the following relations on the set $N \times N$ is an equivalence relation.
    (i) $(a, b)\ R\ (c, d)$ iff $a + d = b + c$
    (ii) $(a, b)\ R\ (c, d)$ iff $ad = bc$
    (iii) $ad(b + c) = bc(a + d)$

22. Let * be a binary operation on set Q defined by $a * b = \frac{ab}{4}$, show that
    (i) 4 is the identify element of * on Q.
    (ii) Every non zero element of Q is invertible with
    $a^{-1} = \frac{16}{a}$, $a \in Q - \{0\}$.

23. Let $f : R \to R$ be defined as $f(x) = 10x + 7$. Find the function $g : R \to R$ such that $gof = fog = I_R$.

H.O.T.S.

VERY SHORT ANSWER TYPE QUESTIONS (1Mark/2 Marks)

24. Let $f : A \to B$ defined as $f(x) = \left(a - x^4\right)^{\frac{1}{4}}$ where $A, B \subset R$, what is fof (x).

25. Let $A = \{1, 2, 3, ..., 9\}$ and R be the relation in $A \times A$ defined by (a, b) R (c, d) if $a + d = b + c$ for $a, b, c, d \in A$. Otain the equivalence class [(2, 5)].

26. Let * be a binary operation, on the set of all non-zero real numbers, given by $a * b = \frac{ab}{5}$ for all $a, b \in R - \{0\}$. Find the value of x, given that $2 * (x * 5) = 10$.

SHORT ANSWER TYPE QUESTIONS (4 Marks)

27. If $f(x) = \frac{x-1}{x+1}$, show that $f(2x) = \frac{3f(x)+1}{f(x)+3}$.

28. Let A $[\{(a, b); a, b \in R$ and $b \neq 0\}$. In A a binary operation '*' is defined as $(a, b) * (c, d) = (ad + bc, bd) \forall (a, b), (c, d) \in A$. Show that A is closed with respect to this composition, * is commutative, associative. Find identity element in A and (if possible) the inverse of (a, b).

29. Let f an Injective function with domain {x, y, z} and range {1, 2, 3} such that exactly one of the following three statements is correct and the remaining two are false.
$f(x) = 1, f(y) \neq 1, f(z) \neq 2$. Find $f^{-1}(1)$.

30. Let $A = Q \times Q$, Q being the set of Rational numbers. Let * be a binary operation on A defined by $(a, b) * (c, d) = (ac, ad + b)$. Show that
(i) * is not commutative
(ii) * is associative
(iii) The identity element w.r.t. * is (1, 0).
(iv) Find the invertible elements of A.

32. Prove that the function $f : N \to N$ defined by $f(x) = x^2 + x + 1$ is one-one, but not onto.

32. If $f : R \to R$ is given by
$f(x) = x^2 + 3x + 1$ and $g : R \to R$ given by $g(x) = 2x - 3$ find (i) fog (ii) gof.

33. If $f : R \to R$ be the function defined by $f(x) = 4x^3 + 7$, show that f is a bijection and find inverse of f.

34. Show that the function $f : R \to \{x \in R$ is one-one and onto function. Let g(x) be the function defined by $g(x) = \sin^{-1} x$. 1st the composition of function gof defined for f and g. Justify. If yes then find gof.

ANSWERS

1. Total number of functions from A to B = $5^3 = 125$.
   Total number of one-one functions from A to B = ${}^5P_3 = 60$.
2. $(5, 7) \notin R$
3. gof = {(1, 3), (3, 1)}.
4. $fog = I_R$ i.e. fog (x) = x.
5. $f^{-1}(x) = \frac{5x+1}{2}$
6. $fof(x) = \frac{x}{2x+1}$
7. (i) 11, (ii) $\frac{1369}{27}$, (iii) –209
8. n!
9. 7.
10. {(a, a), (b, b), (c, c)}
11. 3^4
12. 1
13. e = – 1
14. 2^4
15. –4
16. (a) Neither reflexive nor symmetric nor Transitiive
    (b) Neither reflexive nor symmetric nor Transitiive
    (c) Neither reflexive nor symmetric nor Transitiive
17. (a) Neither one-one nor onto.
    (b) Neither one-one nor onto.
20. (i) * is commutative as well as associative.
    (ii) (1, 1) is identity element.
23. (ii) $g(x) = \frac{x-7}{10}$

ANSWERS OF HOTS

24. x
25. {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}
26. 25

ANSWERS OF HOTS (4 MARKS)

27. Total number of functions from A to B = $5^3 = 125$.
    Total number of one-one functions from A to B = $5P_3 = 60$
28. e = (0, 1), and $(a, b)^{-1} = \left(\frac{-a}{b^2}, \frac{1}{b}\right)$
29. $f^{-1}(1) = y$.
34. Yes, as $g : [-1, 1] \to \left[-\frac{\pi}{2} m \frac{\pi}{2}\right]$, Range (f) $\subseteq$ Domain (g) $gof = \sin^1 \frac{x}{1+|x|}$.

CHAPTER 2 : INVERSE TRIGONOMETRIC FUNCTIONS

ASSIGNMENT
VERY SHORT ANSWER TYPE QUESTIONS (1Mark/2 Marks)

1. Write the principal value of

(i) $\sin^{-1}\left(-\sqrt{3}/2\right)$ (ii) $\sin^{-1}\left(\sqrt{3}/2\right)$

(iii) $\cos^{-1}\left(-\sqrt{3}/2\right)$ (iv) $\cos^{-1}\left(\sqrt{3}/2\right)$

(v) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ (vi) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(vii) $\text{cosec}^{-1}(-2)$ (viii) $\text{cosec}^{-1}(2)$

(ix) $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ (x) $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(xi) $\sec^{-1}(-2)$ (xiii) $\sec^{-1}(2)$

(xiii) $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right) + \tan^{-1}(-1/\sqrt{3})$

2. What is value of the following functions (using principal value).

(i) $\sin^{-1}\left(\sin\frac{4\pi}{5}\right)$ (ii) $\cos^{-1}\left(\cos\frac{7\pi}{5}\right)$

(iii) $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)$ (iv) $\text{cosec}^{-1}\left(\text{cos ec}\frac{3\pi}{4}\right)$

3. Find the value of $\sec^2(\tan^{-1}2) + \text{cosec}^2(\cot^{-1}3)$

4. Prove that $\cos^{-1}\left(\frac{1}{3}\right) = \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) = \tan^{-1}\left(2\sqrt{2}\right)$

5. Write the value of $\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right)$, when $x < 0$.

6. Write the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

7. Evaluate $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$

8. Evaluate $\tan^{-1}\left\{\sin\left(\frac{-\pi}{2}\right)\right\}$

9. Find the value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

10. Find the value of $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$

10. Evaluate $\tan^{-1}\sqrt{3} - \sec(-2)$

12. Evaluate $\sin^{-1}\left[\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right]$

13. Prove that $\tan(\cot^{-1}x) = \cot(\tan^{-1}x)$ state with reason whether the equality is valid for all values of x.

14. Find the value of $\sec\left(\tan^{-1}\frac{y}{2}\right)$

15. Find the value of $\sin\left[2\cot^{-1}\left(\frac{-5}{12}\right)\right]$

16. Evaluate $\cos\left[\sin^{-1}\frac{1}{4}+\sec^{-1}\frac{4}{3}\right]$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

17. Prove than $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)-\cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)=\frac{\pi}{4}\ x\in\left(0,\frac{\pi}{2}\right)$.

18. Prove $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)=\sin^{-1}\frac{x}{a}=\cos^{-1}\left(\frac{\sqrt{a^2-x^2}}{a}\right)$

19. Prove $\cot^{-1}\left[2\tan\left(\cos^{-1}\frac{8}{17}\right)\right]+\tan^{-1}\left[2\tan\left(\sin^{-1}\frac{8}{17}\right)\right]=\tan^{-1}\left(\frac{300}{161}\right)$

20. Solve for x : $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$

21. Solve $\cot^{-1} 2x + \cot^{-1} 3x = \frac{\pi}{4}$

22. Solve $\tan^{-1} 2x - \tan^{-1} 3x = \frac{\pi}{4}$

23. Prove that $\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\frac{\pi}{2}$ Ans. 0, ½

24. Prove that $\tan^{-1}\frac{m}{n}-\tan^{-1}\frac{m-n}{m+n}=\frac{\pi}{4}$

25. Prove that $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]=\frac{x+y}{1-xy}$

26. Solve for x

$$\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)+\frac{1}{2}\tan^{-1}\frac{2x}{1-x^2}=\frac{2\pi}{3}$$

27. Prove that $\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}=\tan^{-1}\frac{63}{16}$

28. Solve for x: $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}\,x = \frac{-\pi}{2}$

29. Solve: $\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$

30. Prove that $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$

31. Prove that $\cos[\tan^{-1}\{\sin(\cos^{-1} x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}$

32. Prove that $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$.

33. Solve $\cos^{-1} x + \sin^{-1}\frac{x}{2}=\frac{\pi}{6}$

34. Prove that $\cot^{-1} 7 + \cot^{-1} g + \cot^{-1} 18 = \cot^{-1} 3$

35. Which is greater than 1 or $\tan^{-1}(1)$?

36. Find the value of $\sin\left(\tan^{-1}\frac{2}{3}\right) + \cos(\tan^{-1}\sqrt{3})$

37. Find the real solutions of the equation $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\left(\sqrt{x}+x+1\right)=\frac{\pi}{2}$

38. Find the simplified form of:

$\cos^{-1}\left(\frac{3}{5}\cos x+\frac{4}{5}\sin x\right)$, where XE$\left[\frac{-3\pi}{4},\frac{\pi}{4}\right]$

39. Find the value of $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}$

40. Show the $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)=\frac{4-\sqrt{7}}{3}$ and justify why the other value $\frac{4-\sqrt{7}}{3}$ is ignored?

41. If a_1, a_2, a_3 ....., an is an arithmetic progression with common difference d, then evaluate the following expression tan

$$\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\tan^{-1}\left(\frac{3}{1+a_3a_4}\right)+...+\tan^{-1}\left(\frac{3}{1+a_na_n}\right)\right]$$

42. Find the real solution of equation $\sqrt{1+\cos 2x}=\sqrt{2}\cos^{-1}(\cos x)$ in$\left[\frac{\pi}{2},\pi\right]$

HOTS

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

43. Prove that $\tan^{-1}\left(\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right)=\tan^{-1}\frac{a}{b}-x$ if $\frac{a}{b}\tan x+1>0$.

44. Prove $\cot\left\{\tan^{-1}x+\tan^{-1}\left(\frac{1}{x}\right)\right\}+\cos^{-1}(1-2x^2)+\cos^{-1}(2x^2-1)=\pi$

45. Show that $2\tan^{-1}\left\{\tan\frac{\alpha}{2},\tan\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right\}$

$=\tan^{-1}\frac{\sin\alpha\cos\beta}{\cos\alpha+\sin\beta}$

46. Prove $\tan^{-1}\left(\frac{a-b}{1+ab}\right)+\tan^{-1}\left(\frac{b-c}{1+bc}\right)+\tan^{-1}\left(\frac{c-a}{1+ac}\right)=0$. If a, b, c > 0

47. Find value of x for

$2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$

48. Express the following in simplex form

$\sin^{-1}[x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}]$

49. If $\tan^{-1}a+\tan^{-1}b+\tan^{-1}c=\pi$ then prove that a + b + c = abc.

50. Find the greatest and least values of $(\sin^{-1} x)^2 + (\cos^{-1} x)^2$.

VERY SHORT ANSWER TYPE QUESTIONS

51. If $f(x) = \cos^{-1}(\log x)$ then what is value of $f(1) + f(e)$

52. What is range of $y = \sin^{-1}[x]$, where [ ] is greatest integer function.

53. What is value of $\cot\left(\sec^{-1} x + \sin^{-1}\frac{1}{x}\right)$

54. Evaluate $\cot(\tan^{-1} a + \cot^{-1} a)$

ANSWERS

1. (i) $-\frac{\pi}{3}$ (ii) $\frac{\pi}{3}$ (iii) $\frac{5\pi}{6}$ (iv) $\frac{\pi}{6}$
   (v) $\frac{-\pi}{6}$ (vi) $\frac{\pi}{6}$ (vii) $\frac{-\pi}{6}$ (viii) $\frac{\pi}{6}$
   (ix) $\frac{-\pi}{3}$ (x) $\frac{\pi}{3}$ (xi) $\frac{2\pi}{3}$ (xii) $\frac{\pi}{3}$
   (xiii) $\frac{\pi}{2}$

2. (i) $\frac{\pi}{5}$ (ii) $\frac{3\pi}{5}$ (iii) $\frac{-\pi}{6}$ (iv) $\frac{\pi}{4}$

3. 15 4. $x = \frac{\sqrt{3}}{2}$ 5. $\frac{\pi}{2}$ 6. $\frac{\pi}{3}$

7. $\frac{1}{\sqrt{10}}$ 8. $\frac{-\pi}{4}$ 9. $\frac{\pi}{6}$ 10. $\frac{-\pi}{8}$

11. $\frac{-\pi}{3}$ 12. $\frac{\pi}{3}$ 13. 14. $\frac{\sqrt{4+y^2}}{2}$

15. $\frac{-120}{169}$ 16. $\frac{3\sqrt{15}-\sqrt{7}}{16}$ 20. $0, \frac{1}{2}$ 21. $x = 1$

22. $x = \frac{1}{6}$ 26. $\sqrt{3}$ 28. $\frac{-1}{12}$ 29. $\frac{3}{4}$

33. 1 35. $\tan 1 > \tan^{-1}(1)$ 36. $\frac{37}{26}$ 37. $0, -1$

38. $\tan^{-1}\frac{4}{3} - x$ 39. $\frac{\pi}{4}$ 41. $\frac{an - a_1}{1 + a_1 an}$ 42. No real solution

47. $x = 0$ or $\frac{\pi}{4}$

48. $\sin^{-1} x - \sin^{-1} \sqrt{x}$

50. $\frac{5\pi^2}{4}, \frac{\pi^2}{8}$

51. $\frac{\pi}{2}$

52. $\left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

53. 0

54. 0

CHAPTER 3-4 : MATRICES AND DETERMINANTS

ASSIGNMENT 1
VERY SHORT ANSWER TYPE QUESTIONS (1Mark/2 Marks)

1. If A is a matrix of order 3X4 such that A'B and AB' are both defined, find the order of B.
2. If A = 2B, where A and B are both square matrices of order 3 and $|A| = 2$. Find $|B|$.
3. Without expanding find the value of the given determinants

(a) $\begin{vmatrix} 42 & 1 & 6 \\ 28 & 7 & 4 \\ 14 & 3 & 2 \end{vmatrix}$ (b) $\begin{vmatrix} 5 & 15 & -25 \\ 7 & 21 & 30 \\ 8 & 24 & 42 \end{vmatrix}$

4. Let A and B be symmetric matrices of same order, shwo that AB – BA is a skew-symmetric matrix.
5. If A is a square matrix of order 3 such that $|adj\ A| = 64$, find $|A|$
6. If B is a skew symmetric matrix, find whether **ABA'** is symmetric or skew symmetric?
7. If A is a matrix of order pxq and both AB and BA are defined. What is the order of B
8. Where $B = \begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & 5 \\ 3 & -5 & 0 \end{bmatrix} = P + Q$, where P is symmetric and Q is skew-symmetric matrix, then find the matrix P
9. If (2, 0), (0, 5) and (a, b) are collinear, find the relation between a and b.
10. If B is a 3×3 matrtix such that $|B| = 15$. Find $|5B|$
11. For what value of x is the matrix $\begin{bmatrix} 3-2x & x-1 \\ 2 & 4 \end{bmatrix}$ singular?
12. If $A = \begin{bmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{bmatrix}$, verify that $A'A = 1$ (identity matrix).
13. If $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$, find x.

14. IF $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ and $|3A| = k|A|$. Find k.

15. Show that (a + b + c) is a factor of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$.

16. Find the area of the triangle whose vertices are A(–3, 5), B(3, –6) and C(7, 2).

17. Write the value of x + y + z

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

18. If $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ find the value of $\frac{x^2 + y^2}{xy}$

19. How many matrices of order 2×3 are possible with each entry as one of the first three prime numbers?

20. Find x, if $\begin{bmatrix} 5 & 3x \\ 2y & z \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 12 & 6 \end{bmatrix}^T$

21. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, find Adj. (Adj. A).

22. If A is a square matrix of order 2 and Adj. (Adj. A) = $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$, find $|A|$.

23. If $A = \begin{bmatrix} 0 & 0 \\ 5 & 0 \end{bmatrix}$ then find $\mathbf{A}^{20}$.

24. If the matrix $\begin{bmatrix} 7 & 2 \\ -k-1 & -k \end{bmatrix}$ is singular, find k.

25. For a matrix $A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ find the value of $|A^{-1}|$.

ANSWERS

1. 3×4
2. $\frac{1}{4}$
3. (a) 0, (b) 0
5. ± 8
6. Skew Symmetric
7. q × p
8. Zero matrix
9. 5a + 2b = 10
10. $5^3 \times 15$
11. x = 1
13. $\pm 2\sqrt{2}$
14. 27
16. $\frac{91}{2}$ sq. units
17. 0
18. Hint: Solve x + 2y = 5, 2x + y = 4 $\Rightarrow$ x = 1, y = 2, $\frac{x^2+y^2}{xy} = \frac{5}{2}$
19. 3^6
20. x = 4
23. $A^{20} = \begin{bmatrix} 0 & 0 \\ 5^{20} & 0 \end{bmatrix}$
24. $k = \frac{2}{5}$

ASSIGNMENT 2
SHORT ANSWER TYPE QUESTIONS (4 MARKS)

1. If $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$ and $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda$

   Then prove that λ = abc

2. If $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$

   $a \neq p, b \neq q, c \neq r$

   then find the value of $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}$

3. If a + b + c = 0 and $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$

then show that, x = 0 or x = $\sqrt{\frac{3}{2}(a^2+b^2+c^2)}$

4. Prove that the value of the determinant $\begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix} = 0$

5. Find the matrix X so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.

6. If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $\mathbf{A}^2 - xA + yI = 0$

7. $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ prove that $\mathbf{A}^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$

8. Given that $A = \begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix}$. Compute $\mathbf{A}^{-1}$ and show that $9I - A = 2A^{-1}$.

9. Given that matrix $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$. Show that $\mathbf{A}^2 - 4A + 71 = 0$. Hence find A^{-1}.

10. Show that $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation $x^2 - 6x + 17 = 0$. Hence find $\mathbf{A}^{-1}$.

11. Prove that the product of two matrices.

$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta s \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ is zero when θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$

12. Given $A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$. Find the product AB and also find $(AB)^{-1}$.

13. Solve the following equations for x.

$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

14. Verify $(AB)^{-1} = B^{-1}A^{-1}$ for the matrices $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$

15. Using matrix method to solve the following system of equations: $5x - 7y = 2$, $7x - 5y = 3$

16. Without expanding show that $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$

17. If $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x & y & z \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$, without expanding prove that $\Delta_1 = \Delta_2$.

18. $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

19. Solve: $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

20. Without expanding evaluate the determinant:

$$\begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (a^y+a^{-y})^2 & (a^y-a^{-y})^2 & 1 \\ (a^2+a^{-2}) & (a^2-a^{-2}) & 1 \end{vmatrix}$$ where $a > 0$ and $x, y, z \in R$.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

21. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = A^2 - 4A + 7I$ Use this result to find A^5.

22. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find adj A and verity that A. (adj A) = (adj A) A = $|A| I_3$.

23. Find the matrix X for which $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} X \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$

24. Using elementary transformations, find the inverse of the matrices.

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

25. The sum of three numbers is 6. If we multiply the third number by 2 and add the first number to the result, we get 7. By adding second and third numbers to three times the first number, we get 12.

26. Solve the system of linear equations by using matrix in equations.

$2x - y + 4z = 1$

$3x - z = 2$

$x - y - 2z = 3$

27. Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, hence solve the system of linear equations:

$x + 2y - 3z = -4$

$2x + 3y + 2z = 2$

$3x - 3y - 4z = 11$

28. The sum of three numbers is 2. If we subtract the second number from twice the first number, we get 3. By adding double the second number and the third number we get 0. Represent it algebraical and find the numbers using matrix method.

29. Compute the inverse of the matrix.

$A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 5 \end{bmatrix}$ and verify that $A^{-1} A = I_3$.

30. If the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ 1 & 0 & 2 \end{bmatrix}$, then compute $(AB)^{-1}$.

31. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations.

$x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$

32. Solve the following system of equations using matrix method.

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$$

$$\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$$

$$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

H.O.T.S. VERY SHORT ANSWER TYPE QUESTIONS (1 MARK/2 MARKS

33. How many matrices of order 2 × 3 are possible with each entry as 0 or 1.

34. If $x \in R, 0 \le x \le \frac{\pi}{2}$, and $\begin{vmatrix} 2\sin x & -1 \\ 1 & \sin x \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ -4 & \sin x \end{vmatrix}$

Then find the value of x.

35. If A is a square matrix of order 3 such that $|\text{adj } A| = 125$, find $|A|$.

36. If $A = \begin{bmatrix} 0 & 0 \\ -3 & 0 \end{bmatrix}$, find A^{20}.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

37. Show that $\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix} = 4abc$

38. Using properties of determinants, show that

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2).$$

39. If x, y, z are the 10th, 13th and 15th terms of a G.P. find the value of $\Delta = \begin{vmatrix} \log x & 10 & 1 \\ \log y & 13 & 1 \\ \log z & 15 & 1 \end{vmatrix}$.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

40. Show that $\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$.

41. If $A = \begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix}$ find A^{-1} and hence solve the equations $3x + 4y + 7z = 14$, $2x - y + 3z = 4$, $x + 2y - 3z = 0$

ANSWERS

5. $x = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

8. $A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

9. $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

10. $A^{-1} = \frac{1}{17}\begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix}$

12. $AB = \begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix}$ $(AB)^{-1} = \frac{1}{6}\begin{bmatrix} 2 & -2 \\ 2 & -1 \end{bmatrix}$

15. $x = \frac{11}{24}, y = \frac{1}{24}$

21. $A^5 = \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$ **(Hint** $A^2 - 4A + 7I = 0$, $A^2 = 4A - 7I$. $A^3 = (4A - 7I)\,A$)

23. $X = \begin{bmatrix} -16 & 3 \\ 24 & -5 \end{bmatrix}$ **(Hint** (if $A \times B = P$, $X = A^{-1}P$) 24. $\frac{1}{5}\begin{bmatrix} -2 & 0 & 3 \\ -1 & 1 & 0 \\ 2 & 1 & -2 \end{bmatrix}$

25. 3, 1, 2

26. $x = \frac{10}{19}, y = \frac{-31}{19}, z = \frac{-8}{19}$

27. $A^{-1} = \frac{-1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$

28. $x = 3, y = -2, z = 1$
$x = -1, y = -2, z = 2$

29. $A^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

30. $(AB)^{-1} = \frac{1}{19}\begin{bmatrix} 16 & 12 & 1 \\ 21 & 11 & -7 \\ 10 & -2 & 3 \end{bmatrix}$ **Hint:** $(AB)^{-1} = B^{-1}A^{-1}$

31. $x = 3, y = -2, z = -1$

32. $x = 2, y = 3, z = 5$ **(Hint** let $\frac{1}{x} = U, \frac{1}{y} = V, \frac{1}{z} = W$ **)**

33. 64

34. $\frac{\pi}{6}, \frac{\pi}{2}$

35. $5\sqrt{5}$

36. 0

39. 0

41. $x = 1, y = 1, z = 1$

CHAPTER 5 : CONTINUITY AND DIFFERENTIABILITY

Assignment 1(Answers)

1. $a = 3, b = -2$
4. $a = 8$
5. $a = \frac{-3}{2}, b \in R - \{0\}, c = \frac{1}{2}$
6. (i) $a = 1, b = -1$
7. $k = -4$
8. $a = \frac{1}{2}$
9. $K = 5$
10. $a = \frac{1}{2}$ $b = 4$
11. $K = 1$
13. (i) $p = -\frac{1}{2}$ (ii) $K = 6$
14. $\frac{1}{2}$
16. $K = \frac{1}{2}$
18. Continuous every where

ASSIGNMENT 2(DIFFERENTIATION)

1. Show that the function f defined by

$$f(x) = \begin{cases} x-1 & \text{if } x < 2 \\ 2x-3 & \text{if } x \geq 2 \end{cases}$$ is not derivable at x = 2.

2. Differentiate $\sqrt{\sin\sqrt{1+x^2}}$ w.r.t. x.
3. Differentiate $\sec(\tan^2\sqrt{x^2+3})$ w.r.t. x.
4. Differentiate $\tan^{-1}\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$ w.r.t. x.
5. Differentiate $\cos^{-1}\left(\frac{3x+4\sqrt{1-x^2}}{5}\right)$ w.r.t. x. (H.O.T.S.)
6. Differentiate $\tan^{-1}\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$ w.r.t. x.

7. Differentiate $\log \sqrt{\frac{1+\cos^2 x}{1-e^{2x}}}$ w.r.t. x.

8. Differentiate $\sin^{-1}\left(\frac{2^{x+1}3^x}{1+(36)^x}\right)$

9. If $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$

10. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, $0 < t < \frac{\pi}{2}$

    Find $\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$

11. Find $\frac{dy}{dx}$ if $(\cos x)^y = (\sin y)^x$

12. Differentiate $\tan^{-1}\frac{x}{1+\sqrt{1+x^2}}$ w.r.t. $\sin\left(2\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)$

13. Differeniate $\text{Sec}^{-1}\frac{1}{2x^2-1}$ w.r.t. $\sqrt{1-x^2}$ at $x = \frac{1}{2}$.

14. Prove that $f(x) = |x + 3|$ is continuous at $x = -3$ but not derivable at $x = -3$.

15. If $3^x + 3^y = 3^{x+y}$ then prove that $\frac{dy}{dx} = -3^{y-x}$.

16. If $y = \tan^{-1} x$ then show that

    $(1 + x^2)\frac{d^2y}{dx^2} + \frac{2xdy}{dx} = 0$

17. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, a being a constant then show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

18. If $y = \tan^{-1}\left(\frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{\sqrt{1+\sin x} + \sqrt{1-\sin x}}\right)$

    Where $\frac{\pi}{2} < x < \pi$ find $\frac{dy}{dx}$.

19. If $x = \sin\left(\frac{1}{a}\log y\right)$ then show that $(1 - x^2)\, y^2 - xy^1 - a^2y = 0$.

20. If $y = \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$ then find $\frac{dy}{dx}$.

21. If $x = ae^{\theta}(\sin\theta - \cos\theta)$,

$y = ae^{\theta}(\sin\theta + \cos\theta)$ then show that $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is 1.

22. If $y = \sin^{-1}\left[\frac{12x + 5\sqrt{1-x^2}}{13}\right]$ find $\frac{dy}{dx}$.

23. If $y = \sin^{-1} x$, find $\frac{d^2y}{dx^2}$ in terms of y.

24. If $y = \left\{x + \sqrt{x^2+1}\right\}^m$

show that $(x^2 + 1)\, y_2 + xy_1 - m^2y = 0$.

25. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, prove that $\frac{dy}{dx} = \frac{-y\log x}{x\log y}$

26. Find $\frac{dy}{dx}$ if $\tan^{-1}(x^2 + y^2) = a$

27. If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$

28. If $y = (\cos x)^{(\cos x)(\cos x)}$, show that $\frac{dy}{dx} = \frac{y^2\tan x}{y\log\cos x - 1}$

29. Verify the Rolle's Theorem for each of the following

(a) $f(x) = \sin^4 x + \cos^4 x$ in $\left[0, \frac{\pi}{2}\right]$

(b) $f(x) = \log(x^2 + 2) - \log 3$ in $[-1, 1]$

30. If $x = \sin t$, $y = \sin pt$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$

31. Using Lagrange's Mean Value Theorem, find a point on the parabola $x = (x - 3)^2$, where the tangent is parallel to the chord joining (3, 0) and (4, 2).

32. It is given that for the function f given by $f(x) = x^3 - 6x^2 + ax + b$, $x \in [1, 3,]$, Rolle's Theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$. Find a and b, if $f(1) = f(3) = 0$.

33. Find the derivative of $f(e^{\tan x})$ with respect to x at x = 0, if f'(0) = 5.

34. If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 1$.

35. If $x = a \sin 2t (1 + \cos 2t)$, $y = b \cos 2t (1 - \cos 2t)$, show that at $t = \frac{\pi}{4}$, $\frac{dy}{dx} = \frac{b}{a}$.

Assignment 2 (Answers)

2. $x\cos\sqrt{1+x^2} / 2\left(\sqrt{\sin\sqrt{1+x^2}}\right)\left(\sqrt{1+x^2}\right)$

3. $\frac{2x}{\sqrt{x^2+3}} \sin\sqrt{x^3+3} \sec^3\sqrt{x^2+3}$

4. $\frac{1}{2\sqrt{1-x^2}}$

5. $\left[\frac{-1}{\sqrt{1-x^2}}\right]$

6. $\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^4}}$

7. $\left[\frac{-1}{2}\tan + \frac{2e^{2x}}{1-e^{2x}}\right]$

8. $\left[\frac{2.6^x \log_e 6}{1+(36)^x}\right]$

9. $\left[\frac{\text{cosec}^4\frac{\theta}{2}}{4a}\right]$

10. $\frac{d^2x}{dt^2} = a(-t\sin t + \cos t)$ $\frac{d^2y}{dt^2} = a(t\cos t + \sin t)$ $\frac{d^2y}{dx^2} = \frac{\sec^3 t}{at}$

11. $\frac{\log(\sin y) + y\tan x}{\log(\cos x) - x\cot y}$

12. $\frac{-\sqrt{1+x^2}}{2(1+x^2)x}$

13. 4

18. $\frac{-1}{2}$

20. $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x-x^2}}$

22. $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

23. $\tan y \sec^2 y$.

26. $\frac{dy}{dx} = \frac{-x}{y}$

31. $\left(\frac{7}{2}, \frac{1}{4}\right)$

32. $a = 11, b = -6$

33. 5